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materiallar to'plami

Uchinchi darajali tenglamalarni echishda talabalarning ijodiy qobiliyatlarini shakllantirish

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Annotatsiya. Bu maqolada uchinchi darajali tenglamalarni echishning umumiy metodi bo'lgan Kardono usuli haqida so'z boradi. Maqola yordamida talabalarda kubik tenglamalarni echish malakalarini shakllantirish, algebra faniga bo'lgan qiziqishlarini orttirish, mustaqil va mantiqiy fikrlarini oshirish ko'zda tutilgan. Maqola yordamida kubik tenglamaning ildizlarini topishda Kardono usulini atroflicha yoritishga harakat qilindi.

Kalit so'zlar: Kubik tenglama, Kardono usuli, kompleks sonlar.

Annotation. This article discusses the Cardono method, a general method for solving cubic equations. With the help of the article, students are expected to develop the skills of solving cubic equations, increase their interest in algebra, and increase their independent and logical thinking. With the help of the article, an attempt was made to cover the Cardono method in finding the roots of a cubic equation.

Key words: Cubic equation, Cardono's method, complex numbers.

Bizga ma'lumki ax+b=0 chiziqli tenglama yagona echimga ega bo'lsa, bu echim $x= -\frac{b}{a}$ ga teng bo'lib, va $ax^2+bx+c=0$ kvadrat tenglamada D>0 katta bo'lsa, uning ildizlari $x_{1,2}=-\frac{b\pm\sqrt{D}}{2a}$ (bu erda $D=b^2-4ac$) ko'rinishida bo'ladi. Tabiiy savol tug'iladi uchinchi darajali tenglamalarni echishning ham umumiy usuli mavjudmi? Albatta, quyida uchinchi darajali tenglamalarni echishning Kardono usuli bilan tanishamiz.

Kubik tenglamani XI asrda buyuk vatandoshimiz Umar Xayyom (1048-1123) birinchi marta geometrik usulda echgan edi. Umar Xayyom 3-darajali tenglama grafigini aylana va parabola tenglamalariga ajralib ularning kesishish nuqtasi berilgan tenglamaning echimi ekanligini isbotlab bergan edi. Italiyalik olim D.Kardono 3-darajali tenglamani echishning o'z usulini 1545-yilda e'lon qilgan. Asosiy qism. 3-darajali tenglamaning umumiy ko'rinish quyidagichadir:

$$ax^3+bx^2+cx+d=0 \qquad (1)$$

(bu erda a,b,c,d lar kompleks sonlar) (1) tenglamaning ikkala tomonini a koeffitsiyentga bo'lib, bosh koeffitsiyenti 1ga teng bo'lgan

$$x^3+bx^2+cx+d=0 \qquad (2)$$

(2) ko'rinishdagi tenglamani hosil qilib olamiz. So'ngra (2) tenglamani soddaroq holga keltirish uchun yangi no'malum y ga nisbatan 2-darajali had qatnashmagan uchinchi darajali tenglamaga keltiramiz. Buning uchun yangi no'malum $y=x+\frac{b}{3}$ bundan) $x=y-\frac{b}{3}$

ko'rinishida bo'lishi lozim ekan. Almashtirishni (2) tenglamamizga olib borib qo'yamiz.

$(y-\frac{b}{3})^3+b(y-\frac{b}{3})^2+c(y-\frac{b}{3})+d=0 \qquad (5)$

(5) tenglamada sodda arifmetik amallarni bajarish natijasida

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$y^3+(c-\frac{b^2}{3})y+(\frac{2b^3}{27}-\frac{bc}{3}+d)=0$
tenglama hosil bo'ldi. b,c va d lar o'zgarmas sonlar bo'lganligi tufayli $p=(c-\frac{b^2}{3})$ $q=(\frac{2b^3}{27}-\frac{bc}{3}+d)$ almashtirish natijasida (5) tenglama

$$y^3+py+q=0 \qquad (6)$$

ko'rinishni oladi.

(6) tenglamaga kubik tenglamaning normal ko'rinishi deyiladi.

(6) Tenglamadagi y o'zgaruvchini α va β kompleks sonlari orqali ifodalaymiz:
$y=\alpha+\beta$
$(\alpha+\beta)^3+(\alpha+\beta)p+q=0$
$\alpha^3+\beta^3+q+(\alpha+\beta)(3\alpha\beta+p)=0 \qquad (7)$

(7) Tenglik bajarilishi uchun $\begin{cases} 3\alpha\beta+p=0 \\ \alpha^3+\beta^3+q=0 \end{cases}$ bo'lishi zarurdir.

Bundan, $\begin{cases} \alpha\beta=-\frac{p}{a} \\ \alpha^3+\beta^3=-q \end{cases}$ ekanligi kelib chiqadi. Demak, $\begin{cases} \alpha^3\beta^3=-\frac{p^3}{27} \\ \alpha^3+\beta^3=-q \end{cases}$ malumki viyet teoremasi yordamida tenglamalar sistemasi orqali ildizlar $\alpha^3\ va\ \beta^3$ bo'lgan kvadrat tenglama hosil qilish mumkin. $t^2+qt-\frac{p^3}{27}=0$ (8)

(8) Tenglamaning ildizlari $\alpha^3\ va\ \beta^3$ ekanligidan $\alpha^3=\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}$ $\beta^3=\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}$ (8) tenglamada, 1-hol. D>0 ya'ni $q^2+\frac{4p^3}{27}>0$ bo'lsa, $\alpha^3\ va\ \beta^3$ larning ikkalasi ham haqiqiy va (6) tenglamaning echimi

$y=\alpha+\beta=\sqrt[3]{\frac{-q+\sqrt{q^2+\frac{4p^3}{27}}}{2}}+\sqrt[3]{\frac{-q-\sqrt{q^2+\frac{4p^3}{27}}}{2}}$ ko'rinishida bo'ladi. Ammo α va β larning 3-darajali kompleks ildizlari 3 tadan bo'lganligi sababli va bu 3tadan kompleks ildizlarni o'rin almashtirib qo'shganimizda jami 9 ta kombinatsiya mavjud bo'ladi. Ammo $y=\alpha+\beta$ yig'indilarning $\alpha\beta=-\frac{p}{27}$ shartni qanoatlantiruvchi juftliklarini olishimiz zarur. sonining α bitta, masalan, α_1 ildizini 1 ning uchinchi darajali ildizlariga ko'paytirishimiz lozim.

$\sqrt[3]{1}=(\cos0+i\sin0)=1$

$$\sqrt[3]{1}=\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i=\varepsilon$$

$$\sqrt[3]{1}=\cos\frac{4\pi}{3}+i\sin\frac{4\pi}{3}=-\frac{1}{2}+\frac{\sqrt{3}}{2}i=\varepsilon^2$$

Demak, (6) tenglamaning echimlari
$y_1=\alpha+\beta$
$y_2=\omega\alpha+\omega^2\beta$
$y_3=\omega^2\alpha+\omega\beta$
va a son y tenglamamizning echimlari esa

$x_1=y_1-\frac{b}{3}=\alpha+\beta-\frac{b}{3}$

$x_2=y_2-\frac{b}{3}=\omega\alpha+\omega^2\beta-\frac{b}{3}$

$x_3=y_3-\frac{b}{3}=\omega^2\alpha+\omega\beta-\frac{b}{3}$

2-hol. (8) tenglamada D=0 bo‘lsa, α^3 va β^3 larning ikkalasi ham haqiqiy (6) tenglamaning ildizlari esa

$y_1=\alpha+\beta$ $x_1=-y_1-\frac{b}{3}$

$y_2=\omega\alpha+\omega^2\beta$ $x_2=-y_2-\frac{b}{3}$

$y_3=\omega^2\alpha+\omega\beta$ $x_3=-y_3-\frac{b}{3}$

3-hol. D<0 ya’ni (8) tenglamada $q^2+\frac{4p^3}{27}<0$ bo‘lsa, u holda α^3 va β^3 larning ikkalasi ham kompleks sonlar bo‘ladi va $\alpha^3=a+ib$ β^3=a-ib ga teng bo‘ladi. Bundan, (6) tenglamaning ildizlari

$y=\alpha+\beta=\sqrt[3]{a+ib}+\sqrt[3]{a-ib}$

$y_1=r(\cos\frac{\alpha}{3}+ i\sin\frac{\alpha}{3})$ $y_2=r(\cos\frac{\alpha+2\pi}{3}+ i\sin\frac{\alpha+2\pi}{3})$ $y_3=r(\cos\frac{\alpha+4\pi}{3}+ i\sin\frac{\alpha+4\pi}{3})$ y ning qiymatlariga mos x lar ham mos ravishda $x=y-\frac{b}{3}$ yordamida topiladi.

Misol: $28x^3-9x^2+1=0$ tenglamani eching.

Yechilishi: Tenglamani $x^3-\frac{9x^2}{28}+\frac{1}{28}=0$ ko‘rinishga keltiramiz.

$x=y-\frac{b}{3}=y-\frac{1}{3}(-\frac{9}{28})=y+\frac{3}{28}$ o‘rniga qo‘yamiz. $(y+\frac{3}{28})^3-\frac{9(y+\frac{3}{28})^2}{28}+\frac{1}{28}=0$ yoki $y^3-\frac{27y}{28}+\frac{730}{28^3}=0$

$P=-\frac{27}{28^2}$ $q=\frac{730}{28^3}$ $y=\alpha+\beta$ deb olamiz. Endi echimlari α^3 va β^3 bo‘lgan kvadrat tenglama tuzib olamiz.

$t^2-\frac{730}{28^3}t-\frac{27^2}{28^6}=0$ $D=(\frac{730}{28^3})^2+4*\frac{27^2}{28^6}>0$ $t_{1,2}=\frac{(-\frac{730}{28^3}\pm\sqrt{(\frac{730}{28^3})^2+4*\frac{27^2}{28^6}})}{2}=\frac{-730\pm728}{2*28^3}$

Bundan, $\alpha^3=\frac{-730+728}{2*28^3}=\frac{2^3}{28^3}$ $\beta^3=\frac{-730-728}{2*28^3}=-\frac{729}{28^3}=-(\frac{9}{28})^3$ $\alpha=-\frac{2}{28}$ $\beta=-\frac{9}{28}$ α va β ma’lum ekanligidan foydalangan holda avval y no‘malumni va u yordamida x ni topib olamiz. Demak, bizga berilgan $28x^3-9x^2+1=0$ tenglamaning $x_1=-\frac{1}{4}$ $x_2=\frac{2+\sqrt{3}i}{7}$ $x_3=\frac{2-\sqrt{3}i}{7}$ ildizlari mavjud.

Xulosa.Uchinchi darajali tenglamalarning,umuman yuqori tartibli tenglamalarning ildizlari matematika fani uchun eng muhim masalalardan bo‘lib kelgan.Siz aziz o‘quvchilarning kubik tenglamani echishdagi bilimlaringiz qiziqishingiz ortgan bo‘lsa bu maqoladan ko‘zlangan asosiy maqsad ham shu edi zero.Ushbu mavzuda olib borilayotgan ilmiy ishlarni davom ettirib,kelgusida bu xususda yaxshi natijalarga erishamiz degan umiddamiz.

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